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ELECTRICAL AND THERMAL CONDUCTIVITIES OF A RELATIVISTIC DEGENERATE PLASMA

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ABSTRACT

The electrical and thermal conductivities are calculated for an electron-proton plasma. The electron distribution function is obtained by solving the Boltzmann equation for a plasma in which the electrons are relativistic and degenerate while the protons are degenerate but nonrelativistic. Such conditions are expected to prevail in neutron-star interiors. The conductivities are increased by factors of order 10^6 over the values obtained when the protons are nondegenerate. The expression obtained for the electrical conductivity differs by a factor 3/4 from the result of a variational calculation by Baym, Pethick, and Pines.
Subject headings: neutron stars — plasmas — relativity

I. INTRODUCTION

The transport properties of a plasma are strongly influenced by particle degeneracy (Kothari 1932*a*, *b*; Marshak 1940; Lee 1950; Mestel 1950; Singwi and Sundaresan 1950; Schatzman 1958; Chiu 1968). At sufficiently high number densities fermion degeneracy results in a relativistic distribution of momenta. The bulk of the charged matter in a neutron-star interior probably comprises an electron-proton plasma in which the electrons are relativistic and degenerate while the protons are degenerate but nonrelativistic (Langer *et al.* 1969; Cameron 1970). The electron-proton degeneracy greatly inhibits Coulomb collisions. The electrical and thermal conductivities—quantities which are inversely proportional to the collision frequency—exhibit corresponding increases.

We consider a degenerate plasma of relativistic electrons and nonrelativistic protons subject to constant magnetic and electric fields and a temperature gradient. The actions of the intense magnetic field (Canuto 1970; Canuto and Solinger 1970; Chiu and Canuto 1970) associated with a neutron star render the plasma anisotropic. In particular, the magnetic field virtually eliminates heat flow perpendicular to the field lines. In spite of the enormous field strength, the electron Fermi energy is much larger than the magnetic quantum of energy. Magnetic quantum effects are therefore minor. Under these conditions it is well known that the transport properties parallel to the magnetic field are independent of the field strength (Kahn and Frederiske 1959). By ignoring the magnetic field term in the Boltzmann equation we obtain scalar transport coefficients. In a more complete analysis these coefficients would appear as diagonal elements of the conductivity tensors.

II. EVALUATION OF TRANSPORT COEFFICIENTS

The theory of thermoelectric phenomena (Sommerfeld 1956) relates the heat flux Q_x and current density J_x to the temperature gradient $\partial T/\partial x$ and the applied electric field E_x,

$$Q_x = -K\frac{\partial T}{\partial x} - (\pi - \psi)J_x\,, \qquad E_x = -\xi\frac{\partial T}{\partial x} + \frac{1}{\sigma}J_x + \frac{\partial\psi}{\partial x}\,.$$

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The quantity ψ is related to the usual electron chemical potential μ_e by $\mu_e = e\psi$, where $-e$ is the electron charge. With this definition, $-\partial\psi/\partial x$ becomes the electric field set up by a gradient in the chemical potential. K is the thermal conductivity, and σ is the electrical conductivity.

The steady-state momentum distribution function for the electrons is assumed to have the form

$$f(\boldsymbol{p}) = f_0(\epsilon) + f_1(\boldsymbol{p}),$$

where $f_0(\epsilon)$ is the Fermi-Dirac distribution, and $f_1(\boldsymbol{p})$ embodies the distorting effects of the field and temperature gradient,

$$f_0(\epsilon) = \{\exp[\beta(\epsilon - \mu_e)] + 1\}^{-1},$$

ϵ is the total electron energy, and $\beta = 1/kT$.

The kinetic expressions for J_x and Q_x are

$$J_x = 2(2\pi\hbar)^{-3}e\int v_x f_1 d^3p\,, \tag{1}$$

$$Q_x = 2(2\pi\hbar)^{-3}\int \epsilon v_x f_1 d^3p\,. \tag{2}$$

The form of $f_1(\boldsymbol{p})$ follows from the steady-state version of the relativistic Boltzmann equation,

$$C(f) = \boldsymbol{v}\cdot\nabla f + e\boldsymbol{E}\cdot\nabla_p f,$$

in which $C(f)$ denotes the collisional rate of exchange of f. In the case at hand $C(f)$ is the rate at which f changes through electron-proton collisions.[1] The evaluation of $C(f)$ is expressed most elegantly in terms of the dynamic form factor, $S(\boldsymbol{\kappa}, \omega)$ (Pines and Nozieres 1966; Baym 1964). The dynamic form factor measures the phase space available to a proton which scatters an electron and recoils, absorbing momentum $\hbar\boldsymbol{\kappa}$ and energy $\hbar\omega$. The steady-state Boltzmann equation becomes

$$C(f_1) = \beta\left[\left(\frac{\epsilon - \mu_e}{T}\right)\frac{\partial T}{\partial x} + \frac{\partial\mu_e}{\partial x} - eE_x\right]v_x f_0(1 - f_0)\,. \tag{3}$$

The details of the evaluation of $C(f_1)$ are sketched in the Appendix. The collision term reduces to the familiar relaxation form

$$C(f_1) = -\nu f_1\,, \tag{4}$$

which yields

$$f_1 = -\frac{\beta}{\nu}\left[\left(\frac{\epsilon - \mu_e}{T}\right)\frac{\partial T}{\partial x} + \frac{\partial\mu_e}{\partial x} - eE_x\right]v_x f_0(1 - f_0)\,. \tag{5}$$

The collision frequency ν is given by

$$\nu = \frac{\epsilon}{2\pi c^2 p^3}\int_{\kappa=0}^{2\kappa_F}\kappa^3|U_\kappa|^2 d\kappa\int_{-\infty}^{+\infty}\frac{S(\kappa, \omega)d\omega}{e^{\beta\hbar\omega} + 1}\,, \tag{6}$$

where U_κ is the Fourier transform of the shielded Coulomb potential. The upper limit

[1] As noted later, the effect of electron-electron and electron-muon collisions may be ignored.

on the κ integration is $2\kappa_F$, where κ_F is the Fermi wavenumber. For degenerate protons the dynamic form factor is (see Appendix)

$$S(\kappa, \omega) = \frac{M^2}{2\pi^2\hbar^3\kappa}\left(\frac{\hbar\omega}{1 - e^{-\beta\hbar\omega}}\right), \tag{7}$$

where M is the proton mass. $|U_\kappa|^2$ is given by

$$|U_\kappa|_2 = \frac{(4\pi e^2)^2(1 - \frac{1}{4}\kappa^2/\kappa_F^2)}{(\kappa^2 + \kappa_{FT}^2)^2}, \tag{8}$$

where κ_{FT} is the Fermi-Thomas wavenumber and is related to the density of states, g, at the Fermi surface by

$$\kappa_{FT}^2 = 4\pi e^2[g_e(\mu_e) + g_p(\mu_p)]. \tag{9a}$$

For relativistic electrons ($\mu_e \gg mc^2$), the shielding is due primarily to the protons,[2]

$$\kappa_{FT}^2 \simeq 4\pi e^2 g_p(\mu_p) = 6\pi Ne^2/\mu_p, \tag{9b}$$

where μ_p is the proton chemical potential

$$\mu_p \simeq \frac{\hbar^2}{2M}(3\pi^2 N)^{2/3},$$

and N is the proton number density. For conditions appropriate to a neutron-star interior, $\kappa_{FT}^2/\kappa_F^2 \ll 1$. Evaluation of the integrals in equation (6) gives, to lowest order in κ_{FT}/κ_F,

$$\nu = \frac{1}{4\kappa_{FT}}\left(\frac{\pi\alpha M}{\hbar\beta}\right)^2 \frac{\epsilon}{p^3}; \qquad \alpha = e^2/\hbar c. \tag{10}$$

The integrals for J_x and Q_x are evaluated with the aid of Sommerfeld's theorem. The electrical conductivity emerges as

$$\sigma = Ne^2 c\tau/\hbar\kappa_F, \tag{11}$$

where N is the electron number density and τ is an appropriately defined mean free time

$$\tau = \frac{4\kappa_{FT}}{c\kappa_F^2}\left(\frac{\hbar^2\kappa_F^2}{\alpha\pi MkT}\right)^2. \tag{12}$$

Baym, Pethick, and Pines (1969) employed a variational method to obtain a lower bound for τ which is three-quarters of the value given by equation (12).

The thermal conductivity may be identified as

$$K = \frac{Nk^2cT}{\hbar\kappa_F}\tau\left(\frac{I_0 I_2 - I_1^2}{I_0}\right), \tag{13}$$

where I_n is an integral over the Fermi-Dirac distribution:

$$I_n = \int f_0(z)dz\frac{d}{dz}\left[z^n\frac{(z^2 - z_1^2)^2}{(z_0^2 - z_1^2)^2}\right];$$

$$z = \epsilon/kT; \qquad z_0 = \mu_e/kT; \qquad z_1 = mc^2/kT. \tag{14}$$

[2] The ratio g_e/g_p is equal to μ_e/Mc^2. For typical neutron-star conditions, $g_e/g_p \sim 1/16$.

In the relativistic degenerate limit

$$I_0 \simeq 1 + 2\pi^2/z_0^2\,, \qquad I_1 \simeq 1 + 10\pi^2/3z_0^2\,, \qquad I_2 \simeq 1 + 5\pi^2/z_0^2\,;$$

and K becomes

$$K = \frac{10\pi^2}{3}\frac{Nk^2cT}{\hbar\kappa_F}\tau\,. \tag{15}$$

Comparing equations (15) and (11) shows that the Wiedemann-Franz law holds, with a Lorenz number

$$\frac{K}{\sigma T} = \frac{10\pi^2}{3}\left(\frac{k}{e}\right)^2\,.$$

For a degenerate but nonrelativistic electron gas the Lorenz number is one-tenth as large (Ziman 1960) having the value $\frac{1}{3}\pi^2(k/e)^2$. Although the Lorenz number shows only a moderate change, the proton degeneracy substantially modifies σ and K. The proton degeneracy increases both σ and K by a factor of order $(\mu_p/kT)^2$ over their values for degenerate relativistic electrons colliding with nondegenerate protons. Typical neutron-star conditions ($N \sim 10^{37}$ cm^{-3}, $T \sim 10^8$ ° K) give $\mu_p \sim 10$ MeV, $kT \sim 10$ keV. The proton-degeneracy enhancement of σ and K is therefore of order 10^6.

The present formulation considers only the effects of electron-proton collisions. With degeneracy sharply reducing the number of protons free to scatter, one is led to consider other sources of resistivity. Electron-electron and electron-muon collisions seem the most obvious candidates. An additional mechanism, the scattering of electrons by the neutron magnetic moment, is favored by the enormous number density of neutrons. Baym *et al.* (1969) have estimated that the scattering of electrons by the neutron magnetic moment is roughly 25 times less effective than Coulomb scattering by the protons.

Consideration of electron-electron collisions shows that they produce negligible changes in σ and K. The dynamic form factor for relativistic electrons can be obtained from equation (7) by replacing M by μ_e/c^2. For typical conditions $\mu_e/Mc^2 \sim 1/16$. The inclusion of electron-electron collisions would thereby modify σ and K by less than 1 percent.

The muons also prove to be an ineffective source of resistivity. Over portions of the interior where the muons are nondegenerate, their number density is too low for them to compete with the protons as electron scatterers. In regions where their number density reaches a respectable level the muons are degenerate, and they lose out to the protons by virtue of the M^2 dependence of $S(\boldsymbol{\kappa}, \omega)$.

III. CONCLUSIONS

The combined effects of degeneracy and shielding reduce the mean electron-proton collision frequency and thereby raise the electrical and thermal conductivities. The enhancement factor is of order $(\mu_p/kT)^2$, which is approximately 10^6 for typical neutron-star conditions.

The physical consequences of the enlarged value of σ have been discussed previously (Baym *et al.* 1969). The enormous thermal conductivity increases the conductive opacity to such an extent that the degenerate interior of a neutron star will be nearly isothermal.

For convenience we express the results for σ, K, and τ in terms of N_{36} (the electron number density in units of 10^{36} cm^{-3}) and T_8 (the temperature in units of 10^8 kelvins):

$$\sigma = 1.6 \times 10^{28} N_{36}{}^{3/2}/T_8{}^2\ \mathrm{s}^{-1}\,; \tag{11a}$$

$$K = 4.2 \times 10^{24} N_{36}{}^{3/2}/T_8{}^2\ \mathrm{ergs\ s^{-1}\ cm^{-1}\ kelvin^{-1}}\,; \tag{15a}$$

$$\tau = 7.4 \times 10^{-15} N_{36}{}^{5/6}/T_8{}^2\ \mathrm{seconds}\,. \tag{12a}$$

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APPENDIX

We sketch here the reduction of the collision integral to the form of equation (14), and the evaluation of $S(\boldsymbol{\kappa}, \omega)$ quoted in equation (7).

Consider a test electron which interacts with a many-body system of scatterers via a scalar potential. Let (μ, m) denote the initial electron (μ) and scatterer (m) states and (ν, n) the final states. A straightforward application of time-dependent perturbation theory gives the transition probability per unit time as

$$\frac{d}{dt}|a_{\nu n}{}^{\mu m}|^2 = \frac{2\pi}{\hbar^2}|U_\kappa|^2|\langle n|\rho_{\boldsymbol{\kappa}}{}^{\dagger}|m\rangle|^2\delta(\Omega) . \tag{A1}$$

U_κ is the Fourier transform of the potential and $\rho_{\boldsymbol{\kappa}}$ is the Fourier transform of the density operator of the system of scatterers (Pines and Nozieres 1966; Baym 1964). As a result of the transition there is a transfer of momentum $\hbar\boldsymbol{\kappa}$ and energy $\hbar\omega$ from the electron to the system of scatterers. In arriving at equation (A1) we have taken plane-wave states for the electron. The δ-function ensures energy conservation, with

$$\hbar\Omega = E_n - E_m - \hbar\omega ,$$

where E_m and E_n denote the initial and final energies of the scattering system. The quantity given by equation (A1) is the probability per unit time of a transition from an "*occupied*" initial state (μ, m) into an "*empty*" final state (ν, n). To construct a "collision integral," we must introduce various occupation probabilities, sum over the final states of the test particle, and sum over the initial and final states of the scattering system. Write

$$W_m = \text{probability that the initial scatterer state is occupied;}$$

$$f(p_\mu) \equiv f_\mu = \text{probability that the initial electron state is occupied .}$$

Then, $W_m f_\mu(1 - f_\nu)$ is the probability that the initial state is occupied and that the final electron state is open, and $W_m f_\mu(1 - f_\nu)d|a_{\nu n}{}^{\mu m}|^2/dt$ is the expected rate of (μ, m) into (ν, n). Summing this over all ν, m, and n gives $\mathscr{L}(f_\mu)$, the total rate at which the test electron state μ is depopulated:

$$\mathscr{L}(f_\mu) = 2\pi/\hbar^2 \sum_{\nu,m,n} |U_\kappa|^2|\langle n|\rho_{\boldsymbol{\kappa}}{}^{\dagger}|m\rangle|^2\delta(\Omega)W_m f_\mu(1 - f_\nu) .$$

The mutual dependence of ω and $\boldsymbol{\kappa}$ is contained both in $\delta(\Omega)$ and in the δ-function which appears in the density of final electron states,

$$\sum_\nu = (2\pi\hbar)^{-3}\int d^3p_\nu d\omega\delta[\omega - \hbar^{-1}(\epsilon_\mu - \epsilon_\nu)] .$$

From the conservation of momentum, $\boldsymbol{p}_\nu = \boldsymbol{p}_\mu - \hbar\boldsymbol{\kappa}$, it follows that (for a given $\boldsymbol{p}_\mu$) $d^3p_\nu/\hbar^3 = d^3\kappa$, whence

$$\sum_\nu = (2\pi)^{-3}\int d^3\kappa d\omega\delta(s)$$

with $\hbar s = \hbar\omega - (\epsilon_\mu - \epsilon_\nu)$.

The dynamic form factor for the scatterers is defined as

$$S(\varkappa, \omega) = \sum_{m,n} W_m |\langle n|\rho_\varkappa^\dagger|m\rangle|^2 \delta(\Omega)\,, \tag{A2}$$

whereupon $\mathscr{L}(f_\mu)$ becomes

$$\mathscr{L}(f_\mu) = (2\pi\hbar)^{-3} \int d^3\kappa d\omega \delta(s)|U_\kappa|^2 S(\varkappa, \omega) f_\mu (1 - f_\nu)\,.$$

The "inverse" interactions which populate the electron state of momentum $\boldsymbol{p}_\mu$ do so at a rate

$$\mathscr{E}(f_\mu) = (2\pi\hbar)^{-3} \int d^3\kappa d\omega \delta(s)|U_\kappa|^2 S(-\varkappa, -\omega) f_\nu (1 - f_\mu)\,.$$

The total rate of change in $f(\boldsymbol{p})$ is evidently $\mathscr{E}(f) - \mathscr{L}(f)$, which we designate $C(f)$:

$$C(f) = (2\pi\hbar)^{-3} \int d^3\kappa d\omega \delta(s)|U_\kappa|^2 [S(-\varkappa, -\omega) f_\nu (1 - f_\mu) - S(\varkappa, \omega) f_\mu (1 - f_\nu)]. \tag{A3}$$

By making use of the second quantized representation of $\rho_\varkappa$, the dynamic form factor for a gas of noninteracting fermions may be reduced to (Pines and Nozieres 1966)

$$S(\varkappa, \omega) = \sum_{\boldsymbol{P}} F(\boldsymbol{P})\,[1 - F(\boldsymbol{P} + \hbar\varkappa)]\delta(\Omega)\,, \tag{A4}$$

in which $F(\boldsymbol{P})$ is the occupation probability for a scatterer of momentum $\boldsymbol{P}$. One readily verifies from the principle of detailed balance [according to which $C(f) = 0$ when $f = f_0(\epsilon)$],

$$S(-\varkappa, -\omega) = e^{-\beta\hbar\omega} S(\varkappa, \omega)\,.$$

If next we write the perturbed portion of $f_1(\boldsymbol{p})$ as

$$f_1(\boldsymbol{p}) = \boldsymbol{p}\cdot\boldsymbol{h}(\epsilon) f_0 (1 - f_0)\,, \tag{A5}$$

then the collision integral reduces to

$$C(f_1) = (2\pi\hbar)^{-3} \int d^3\kappa d\omega \delta(s)|U_\kappa|^2 S(\varkappa, \omega) f_0(\epsilon)[1 - f_0(\epsilon')][\boldsymbol{p}'\cdot\boldsymbol{h}(\epsilon') - \boldsymbol{p}\cdot\boldsymbol{h}(\epsilon)] \tag{A6}$$

with $\epsilon' = \epsilon - \hbar\omega$, $\boldsymbol{p}' = \boldsymbol{p} - \hbar\varkappa$. The factor $f_0(\epsilon)[1 - f_0(\epsilon')]$ may be written as

$$f_0(\epsilon)[1 - f_0(\epsilon')] = f_0(\epsilon)[1 - f_0(\epsilon)]\left(1 + \frac{e^{-\beta\hbar\omega} - 1}{e^{\eta - \beta\hbar\omega} + 1}\right)\,,$$

where $\eta = \beta(\epsilon - \mu_e)$. The eventual integration over ϵ (to obtain the transport coefficients) is dominated by the factor $f_0(\epsilon)[1 - f_0(\epsilon)]$, which is sharply peaked at $\epsilon = \mu_e$, i.e., at $\eta = 0$. We therefore take $\eta = 0$ in the last factor, obtaining

$$f_0(\epsilon)[1 - f_0(\epsilon')] \simeq f_0(\epsilon)[1 - f_0(\epsilon)] \frac{2}{e^{\beta\hbar\omega} + 1}\,.$$

Further, we write

$$\boldsymbol{p}'\cdot\boldsymbol{h}(\epsilon') - \boldsymbol{p}\cdot\boldsymbol{h}(\epsilon) \simeq (\boldsymbol{p}' - \boldsymbol{p})\cdot\boldsymbol{h}(\epsilon)\,,$$

and make use of the degeneracy restrictions again to obtain

$$\int d^3\kappa \cdots (\boldsymbol{p}' - \boldsymbol{p})\cdot\boldsymbol{h}(\epsilon) \simeq -\boldsymbol{p}\cdot\boldsymbol{h}(\epsilon)\int d^3\kappa \cdots \frac{(\hbar\kappa)^2}{2p^2}\,.$$

These steps leave the collision operator in the form of equation (4), $C(f_1) = -\nu f_1(\boldsymbol{p})$, with

$$\nu = \frac{1}{4\pi^2 p^2}\int \kappa^2 d^3\kappa d\omega\delta(s)|U_\kappa|^2 \frac{S(\kappa, \omega)}{e^{\beta\hbar\omega}+1}\,. \tag{A7}$$

The integration over the δ-function is performed by letting $\boldsymbol{p}$ define the z-axis for the $d^3\kappa$ integration. Writing

$$d^3\kappa = \kappa^2 d\kappa \sin\theta d\theta d\phi$$

and noting

$$(\epsilon')^2 = \epsilon^2 + (\hbar\kappa c)^2 - 2\hbar\kappa p c^2 \cos\theta$$

and

$$\int \delta[f(x) - f(x_0)]dx = \frac{1}{|df/dx|_{x=x_0}}$$

produces

$$\int \sin\theta d\theta\delta(s) \simeq \frac{\epsilon}{\kappa p c^2}\,,$$

and leaves the desired form of the collision frequency ν:

$$\nu = \frac{\epsilon}{2\pi c^2 p^3}\int_{\kappa=0}^{2\kappa_F} \kappa^3 d\kappa |U_\kappa|^2 \int_{-\infty}^{+\infty} \frac{d\omega S(\boldsymbol{\kappa}, \omega)}{e^{\beta\hbar\omega}+1} \tag{6}$$

To convert the expression for $S(\boldsymbol{\kappa}, \omega)$ given by equation (A4) into the result quoted in equation (7), the δ-function is first removed by an integration over the polar angle. This gives

$$S(\boldsymbol{\kappa}, \omega) = \frac{\omega}{2\pi^2\hbar^3 c^2}\int EPdPF(E)[1 - F(E+\hbar\omega)]\,.$$

For scatterers which are degenerate but nonrelativistic fermions of mass M_s this becomes

$$S(\boldsymbol{\kappa}, \omega) = \frac{M_s^2}{2\pi^2\hbar^3 c^2 \kappa\beta}\int_{\eta_0}^{+\infty} d\eta \frac{1}{e^\eta + 1}\frac{1}{e^{-\eta-\beta\hbar\omega}+1}\,,$$

$$\beta\eta_0 \simeq [\tfrac{1}{2}M_s(\omega/\kappa)^2 - \mu]\,.$$

The familiar degeneracy condition is $\beta\mu \gg 1$ which translates into the following numerical constraint on N_s, the number density of the scatterers:

$$N_s \gg 3 \times 10^{32}(M_s/M)^{3/2} T_8^{3/2}\ \text{cm}^{-3}\,, \tag{A8}$$

where M is the proton mass.

In arriving at the form of $S(\boldsymbol{\kappa}, \omega)$ given below (eq. [7]) we take $\eta_0 = -\infty$; hence, it is also necessary to satisfy

$$\tfrac{1}{2}M_s(\omega/\kappa)^2 < \mu$$

for the important ranges of ω and κ. Taking $\omega = kT/\hbar$ and $\kappa = \kappa_{FT}$ leads to

$$N_s > 3 \times 10^{31}(M_s/M)T_8^2 \text{ cm}^{-3} . \tag{A9}$$

Under conditions envisioned for neutron-star interiors, both equations (A8) and (A9) are amply satisfied for protons and muons. The M_s^2 dependence of $S(\varkappa, \omega)$, and thus ν, permits electron-muon scattering to be ignored in favor of electron-proton scattering. Taking $\eta_0 = -\infty$ and noting

$$\int_{-\infty}^{+\infty} d\eta \frac{1}{e^{\eta} + 1} \frac{1}{e^{-\eta-\beta\hbar\omega} + 1} = \frac{\beta\hbar\omega}{1 - e^{-\beta\hbar\omega}}$$

results in

$$S(\varkappa, \omega) = \frac{M_s^2}{2\pi^2\hbar^3\kappa} \frac{\hbar\omega}{1 - e^{-\beta\hbar\omega}} . \tag{A10}$$

With $M_s = M$, equation (A10) reduces to the result quoted in equation (7). In the case where the fermions are degenerate and relativistic the dynamic form factor is given by equation (A10) with M_s^2 replaced by $(\mu/c^2)^2$, where μ is the chemical potential.

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